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Light-cone distribution amplitudes for the light 1^1P_1 mesons

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Abstract: We present a study of light-cone distribution amplitudes of the light 1^1P_1 mesons. The first few Gegenbauer moments of leading twist light-cone distribution amplitudes are calculated by using the QCD sum rule technique.

Keywords: QCD, Nonperturbative Effects, Sum Rules.

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Contents

1. Introduction

In the QCD description of various exclusive processes, it is necessary to know the hadronic wave functions in terms of light-cone distribution amplitudes (LCDAs). The role of LCDAs is analogous to that of parton distributions in inclusive processes. The conformal properties for multiplicative renormalizability of a nonlocal operator, from which the defined wave functions are written as a sum of LCDAs with specific conformal spins in the asymptotic limit, have been systematically studied [1, 2]. For each conformal spin, the dependence of the distribution amplitudes on the transverse coordinates is governed by the renormalization group equation and the dependence on the longitudinal coordinates is involved in "spherical harmonics" of the $\mathrm{SL}(2,\mathbb{R})$ group. The conformal invariance of QCD guarantees that for leading twist LCDAs there is no mixing among Jacobi polynomials of different spins to leading logarithmic accuracy, and, moreover, the anomalous dimensions are ordered with conformal spin.

In the present work, we devote to the study of leading twist LCDAs of 1^1P_1 mesons: $b_1(1235), h_1(1170), h_1(1380)$, and K_{1B}. We give a detailed calculations for Gegenbauer moments of leading twist LCDAs. $h_1(1380)$ is a 1^1P_1 meson[1] and its properties are not experimentally well-established [3]. The quark content of $h_1(1380)$ was suggested as $\bar{s}s$ in the QCD sum rule calculation [4]. K_{1B} are the 1^1P_1 isodoublet strange states. K_{1B} and K_{1A} (a 1^3P_1 state) are the mixtures of the real physical states $K_1(1270)$ and $K_1(1400)$, where the mixing angle may be close to 45° [3].

[1] $h_1(1380)$ with $I^G(J^{PC}) = ?^-(1^{+-})$ was denoted as H' in old classification. Its isospin may be 0, but not confirmed yet.

In the quark model, the 1^1P_1 meson is represented as a constituent quark-antiquark pair with total spin $S = 0$ and angular momentum $L = 1$. Nevertheless, a real hadron in QCD language should be described in terms of a set of Fock states for which each state has the same quantum number as the hadron, and the leading twist LCDAs are thus interpreted as amplitudes of finding the meson in states with a minimum number of partons. Due to the G-parity, the leading twist LCDA $\Phi_\parallel^A$ of a 1^1P_1 meson defined by the nonlocal axial-vector current is antisymmetric under the exchange of *quark* and *anti-quark* momentum fractions in the SU(3) limit, whereas the leading twist LCDA $\Phi_\perp^A$ defined by the nonlocal tensor current is symmetric. The large magnitude of the first Gegenbauer moment of $\Phi_\parallel^A$ could have a large impact on the longitudinal fraction of factorization-suppressed B decays involving a 1^1P_1 meson evaluated in the QCD factorization framework [5]. Furthermore, $\Phi_\perp^A$ is relevant not only for exploring the tensor-type new-physics effects in B decays [5] but also for $B \to 1^1P_1\,\gamma$ studies.

2. Two-parton distribution amplitudes of $^1P_1$ axial vector mesons

We restrict ourselves to the two-parton LCDAs of the $^1P_1$ axial vector mesons, denoted as A here.[2] Throughout the present paper, we define $z = y - x$ with $z^2 = 0$, and introduce the light-like vector $p_\mu = P_\mu - m_A^2 z_\mu/(2P\cdot z)$ with the meson's momentum $P^2 = m_A^2$. The meson polarization vector $\epsilon_\mu^{(\lambda)}$ is decomposed into *longitudinal* and *transverse projections*, defined as [6, 7]

$$\epsilon_{\parallel\,\mu}^{(\lambda)} \equiv \frac{\epsilon^{(\lambda)}\cdot z}{P\cdot z}\left(P_\mu - \frac{m_A^2}{P\cdot z} z_\mu\right), \qquad \epsilon_{\perp\,\mu}^{(\lambda)} = \epsilon_\mu^{(\lambda)} - \epsilon_{\parallel\,\mu}^{(\lambda)}, \tag{2.1}$$

respectively. In QCD description of hard processes involving axial vector mesons, one encounters bilocal operators sandwiched between the vacuum and the meson,

$$\langle A(P,\lambda)|\bar q_1(y)\Gamma[y,x]q_2(x)|0\rangle \tag{2.2}$$

where Γ is a generic notation for the Dirac matrix structure and the path-ordered gauge factor is

$$[y,x] = \mathrm{P}\exp\left[ig_s\int_0^1 dt\,(x-y)_\mu A^\mu(tx+(1-t)y)\right]. \tag{2.3}$$

This factor is equal to unity in the light-cone gauge which is equivalent to the fixed-point gauge (or called the Fock-Schwinger gauge) $(x-y)^\mu A_\mu(x-y) = 0$ as the quark-antiquark pair is at the light-like separation. For simplicity, here and below we do not show the gauge factor. For $b_1(1235)$ $[h_1(1170)]$, the operator in eq. (2.2) corresponds to $1/\sqrt{2}(\bar u(y)\Gamma u(x) - [+]\bar d(y)d(x))$. In the present study, we adopt the conventions: $D_\alpha = \partial_\alpha + ig_s A_\alpha^a\lambda^a/2$, $\widetilde G_{\alpha\beta} = (1/2)\epsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$, $\epsilon^{0123} = -1$.

[2]If the $^1P_1$ particle is made of $\bar q q$, then its charge conjugate C is -1, i.e., $J^{PC} = 1^{+-}$.

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2.1 Definitions

In general, the LCDAs are scheme- and scale-dependent. The chiral-even LCDAs are given by

$$\langle A(P,\lambda)|\bar q_1(y)\gamma_\mu\gamma_5 q_2(x)|0\rangle = \tag{2.4}$$
$$= i f_A m_A \int_0^1 du\, e^{i(u\,py+\bar u px)} \left\{ p_\mu \frac{\epsilon^{*(\lambda)}z}{pz}\,\Phi_\parallel(u) + \epsilon^{*(\lambda)}_{\perp\mu}\, g_\perp^{(v)}(u) - \frac{1}{2} z_\mu \frac{\epsilon^{*(\lambda)}z}{(pz)^2} m_A^2 g_3(u) \right\},$$
$$\langle A(P,\lambda)|\bar q_1(y)\gamma_\mu q_2(x)|0\rangle =$$
$$= -i f_A\, m_A \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}_{(\lambda)} p^\rho z^\sigma \times \int_0^1 du\, e^{i(u\,py+\bar u\,px)}\, \frac{g_\perp^{(a)}(u)}{4}, \tag{2.5}$$

with the matrix elements involving an odd number of γ matrices and and $\bar u \equiv 1-u$. The chiral-odd LCDAs are given by

$$\langle A(P,\lambda)|\bar q_1(y)\sigma_{\mu\nu}\gamma_5 q_2(x)|0\rangle =$$
$$= f_A^\perp \int_0^1 du\, e^{i(u\,py+\bar u\,px)} \left\{ (\epsilon^{*(\lambda)}_{\perp\mu} p_\nu - \epsilon^{*(\lambda)}_{\perp\nu} p_\mu)\Phi_\perp(u) + \frac{m_A^2\,\epsilon^{*(\lambda)}z}{(pz)^2}(p_\mu z_\nu - p_\nu z_\mu)\, h_\parallel^{(t)}(u) + \right.$$
$$\left. + \frac{1}{2}(\epsilon^{*(\lambda)}_{\perp\mu} z_\nu - \epsilon^{*(\lambda)}_{\perp\nu} z_\mu)\frac{m_A^2}{p\cdot z} h_3(u,\mu^2) \right\}, \tag{2.6}$$
$$\langle A(P,\lambda)|\bar q_1(y)\gamma_5 q_2(x)|0\rangle =$$
$$= f_A^\perp\, m_A^2 \epsilon^{*(\lambda)} z \int_0^1 du\, e^{i(u\,py+\bar u\,px)}\, \frac{h_\parallel^{(s)}(u)}{2}, \tag{2.7}$$

with the matrix elements having an even number of γ matrices. Here the LCDAs $\Phi_\parallel, \Phi_\perp$ are of twist-2, $g_\perp^{(v)}, g_\perp^{(a)}, h_\perp^{(t)}, h_\parallel^{(s)}$ of twist-3, and g_3, h_3 of twist-4. In SU(3) limit, due to G-parity, $\Phi_\parallel, g_\perp^{(v)}$, $g_\perp^{(a)}$, and g_3 are antisymmetric under the replacement $u \to 1-u$, whereas $\Phi_\perp, h_\parallel^{(t)}$, $h_\parallel^{(s)}$, and h_3 are symmetric. Owing to the above properties, we therefore normalize the distribution amplitudes to be subject to

$$\int_0^1 du \Phi_\perp(u) = 1, \tag{2.8}$$

and take $f_A = f_A^\perp(\mu = 1\,\mathrm{GeV})$ in the study. Note that

$$\int_0^1 du \Phi_\parallel(u) = \int_0^1 du g_\perp^{(a)}(u) = \int_0^1 du g_\perp^{(v)}(u) = \int_0^1 du g_3(u) = 0 \tag{2.9}$$

in SU(3) limit (see footnote 3 for further discussions). We will not further discuss g_3 and h_3 below.

2.2 Chiral-even light-cone distribution amplitudes

$\Phi_\parallel(u,\mu)$ can be expanded in a series of Gegenbauer polynomials [1, 2]:

$$\Phi_\parallel(u,\mu) = 6u(1-u)\sum_{l=0}^{\infty} a_l^\parallel(\mu) C_l^{3/2}(2u-1), \tag{2.10}$$

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where μ is the normalization scale and the multiplicatively renormalizable coefficients (or called Gegenbauer moments) are:

$$a_l^{\parallel}(\mu) = \frac{2(2l+3)}{3(l+1)(l+2)} \int_0^1 dx\, C_l^{3/2}(2x-1)\Phi_{\parallel}(x,\mu)\,. \tag{2.11}$$

In the limit of $m_{q1} = m_{q2}$, only terms with odd l survive due to G-parity invariance. In the expansion of $\Phi_{\parallel}(u,\mu)$ in eq. (2.10), the conformal invariance of the light-cone QCD exhibits that partial waves with different conformal spin cannot mix under renormalization to leading-order accuracy. As a consequence, the Gegenbauer moments $a_l^{\parallel}$ renormalize multiplicatively:

$$a_l^{\parallel}(\mu) = a_l^{\parallel}(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\gamma_{(l)}^{\parallel}/b}, \tag{2.12}$$

where $b = (11N_c - 2n_f)/3$ and the one-loop anomalous dimensions are [8]

$$\gamma_{(l)}^{\parallel} = C_F\left(1 - \frac{2}{(l+1)(l+2)} + 4\sum_{j=2}^{l+1}\frac{1}{j}\right), \tag{2.13}$$

with $C_F = (N_c^2 - 1)/(2N_c)$.

Applying the QCD equations of motion, discussed in detail in refs. [9, 1, 6, 7], one can obtain some useful nonlocal operator identities such that the two-parton disctribution amplitudes $g_{\perp}^{(a)}$ and $g_{\perp}^{(v)}$ can be represented in terms of $\Phi_{\perp,\parallel}$ and two three-parton distribution amplitudes. Neglecting three-parton distribution amplitudes containing gluons and terms proportional to light quark masses, $g_{\perp}^{(v)}$ and $g_{\perp}^{(a)}$ are thus related to the twist-2 one by Wandzura-Wilczek–type relations:

$$g_{\perp}^{(v)WW}(u) \simeq \frac{1}{2}\left[\int_0^u dv\,\frac{1}{\bar{v}}\Phi_{\parallel}(v) + \int_u^1 dv\,\frac{1}{v}\Phi_{\parallel}(v)\right], \tag{2.14}$$

$$g_{\perp}^{(a)WW}(u) \simeq 2\bar{u}\int_0^u dv\,\frac{1}{\bar{v}}\Phi_{\parallel}(v) + 2u\int_u^1 dv\,\frac{1}{v}\Phi_{\parallel}(v)\,. \tag{2.15}$$

2.3 Chiral-odd light-cone distribution amplitudes

The leading twist LCDAs $\Phi_{\perp}^{A}(u,\mu)$ can be expanded as [1, 2]

$$\Phi_{\perp}^{A}(u,\mu) = 6u(1-u)\left[1 + \sum_{l=1}^{\infty} a_l^{\perp,A}(\mu) C_l^{3/2}(2u-1)\right], \tag{2.16}$$

where the multiplicatively renormalizable Gegenbauer moments, in analogy to eq. (2.11), read

$$a_l^{\perp,A}(\mu) = \frac{2(2l+3)}{3(l+1)(l+2)} \int_0^1 dx\, C_l^{3/2}(2x-1)\Phi_{\perp}^{A}(x,\mu)\,, \tag{2.17}$$

which satisfy

$$a_l^{\perp,A}(\mu) = a_l^{\perp,A}(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-(\gamma_{(l)}^{\perp} - \frac{4}{3})/b}, \tag{2.18}$$

with the one-loop anomalous dimensions being [8]

$$\gamma_{(l)}^{\perp} = C_F \left(1 + 4\sum_{j=2}^{l+1}\frac{1}{j}\right). \tag{2.19}$$

$a_l^{\perp,A}$ vanish in the SU(3) limit when l are odd.

Using the equations of motion given in refs. [9, 6], the two-parton distribution amplitudes $h_\parallel^{(t)}$ and $h_\parallel^{(s)}$ can be represented in terms of $\Phi_{\perp,\parallel}$ and a three-parton distribution amplitude. The two-parton twist-3 distribution amplitudes are thus related to the twist-2 one approximately by Wandzura-Wilczek–type relations

$$h_\parallel^{(t)WW}(u) = \xi\left(\int_0^u dv\frac{\Phi_\perp(v)}{\bar{v}} - \int_u^1 dv\frac{\Phi_\perp(v)}{v}\right), \tag{2.20}$$
$$h_\parallel^{(s)WW}(u) = 2\left(\bar{u}\int_0^u dv\frac{\Phi_\perp(v)}{\bar{v}} + u\int_u^1 dv\frac{\Phi_\perp(v)}{v}\right). \tag{2.21}$$

3. The tensor couplings and Gegenbauer moments

3.1 Input parameters

We calculate renormalization-group (RG) improved QCD sum rules [10] of the tensor couplings and Gegenbauer moments for the 1^1P_1 mesons. We employ the SU(2) flavor symmetry, i.e., do not distinguish between $b_1(1235)$ and $h_1(1170)$. We therefore simply use b_1 to denote $b_1(1235)$ and $h_1(1170)$, and h_1 to stand for the $h_1(1370)$ meson. In the numerical analysis we take into account $\alpha_s(1\text{ GeV}) = 0.517$, corresponding to the world average $\alpha_s(m_Z) = 0.1213$ [3], and the following relevant parameters at the scale $\mu = 1\,\text{GeV}$ [11]:

$$\begin{aligned}
\langle \alpha_s G^a_{\mu\nu}G^{a\mu\nu}\rangle &= \frac{0.474\,\text{GeV}^4}{4\pi}, \\
\langle \bar{u}u\rangle \cong \langle \bar{d}d\rangle &= -(0.24\pm 0.005)^3\,\text{GeV}^3, & \langle \bar{s}s\rangle &= 0.8\langle \bar{u}u\rangle, \\
\frac{(m_u+m_d)}{2} &= 5\,\text{MeV}, & m_s &= 120\,\text{MeV}, \\
\langle g_s\bar{u}\sigma Gu\rangle \cong \langle g_s\bar{d}\sigma Gd\rangle &= -0.8\langle \bar{u}u\rangle, & \langle g_s\bar{s}\sigma Gs\rangle &= 0.8\langle g_s\bar{u}\sigma Gu\rangle,
\end{aligned} \tag{3.1}$$

with the corresponding anomalous dimensions of operators satisfying [11]:

$$\begin{aligned}
m_{q,\mu} &= m_{q,\mu_0}\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\frac{4}{b}}, \\
\langle \bar{q}q\rangle_\mu &= \langle \bar{q}q\rangle_{\mu_0}\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{\frac{4}{b}}, \\
\langle g_s\bar{q}\sigma\cdot Gq\rangle_\mu &= \langle g_s\bar{q}\sigma\cdot Gq\rangle_{\mu_0}\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\frac{2}{3b}}, \\
\langle \alpha_s G^2\rangle_\mu &= \langle \alpha_s G^2\rangle_{\mu_0}.
\end{aligned} \tag{3.2}$$

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We adopt the vacuum saturation approximation for describing the four-quark condensates, i.e.,

$$\langle 0|\bar{q}\Gamma_i\lambda^a q\bar{q}\Gamma_i\lambda^a q|0\rangle = -\frac{1}{16N_c^2}\mathrm{Tr}(\Gamma_i\Gamma_i)\mathrm{Tr}(\lambda^a\lambda^a)\langle\bar{q}q\rangle^2\,, \tag{3.3}$$

and neglect the possible effects due to their anomalous dimensions.

3.2 The tensor couplings for 1^1P_1 mesons

To determine the the tensor couplings of the 1^1P_1 Mesons, A, defined as

$$\langle 0|\bar{q}_2\sigma_{\mu\nu}q_1|A(P,\lambda)\rangle = i f_A^\perp\,\epsilon_{\mu\nu\alpha\beta}\epsilon^\alpha_{(\lambda)}P^\beta\,, \tag{3.4}$$

i.e.,

$$\langle 0|\bar{q}_2\sigma^{\mu\nu}\gamma_5 q_1|A(P,\lambda)\rangle = -f_A^\perp(\epsilon^\mu_{(\lambda)}P^\nu - \epsilon^\nu_{(\lambda)}P^\mu)\,, \tag{3.5}$$

we consider the correlation function of two tensor currents:

$$\Pi_{\mu\nu\alpha\beta} = i\int d^4x\, e^{iqx}\,\langle 0|T[\bar{q}_1(x)\sigma_{\mu\nu}q_2(x)\ \bar{q}_2(0)\sigma_{\alpha\beta}q_1(0)]|0\rangle\,, \tag{3.6}$$

where $\Pi_{\mu\nu\alpha\beta}$ can be decomposed into two Lorentz invariant functions $\Pi^\pm$ as

$$\begin{aligned}\Pi_{\mu\nu\alpha\beta} = {} & [g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}]\Pi^+(q^2) + \\ & +[g_{\mu\beta}q_\nu q_\alpha + g_{\nu\alpha}q_\mu q_\beta - g_{\mu\alpha}q_\nu q_\beta - g_{\nu\beta}q_\mu q_\alpha]\frac{\Pi^+(q^2)+\Pi^-(q^2)}{q^2}\,.\end{aligned} \tag{3.7}$$

The $\Pi^{+,(-)}(q^2)$, corresponding to interpolating states with positive (negative) parity, respectively, were computed in [4]:

$$\begin{aligned}\Pi^\mp(q^2) = {} & -\frac{1}{8\pi^2}q^2\ln\frac{-q^2}{\mu^2}\left[1\pm 6\frac{m_1m_2}{q^2}+\frac{\alpha_s}{3\pi}\left(\ln\frac{-q^2}{\mu^2}+\frac{7}{3}\right)\right]+ \\ & +\frac{1}{q^2}\left[-\frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle+\left(\frac{1}{2}m_1\pm m_2\right)\langle\bar{q}_1q_1\rangle+\left(\frac{1}{2}m_2\pm m_1\right)\langle\bar{q}_2q_2\rangle\right]- \\ & -\frac{1}{q^4}\left[\frac{1}{6}\left(2m_1\pm m_2\right)\langle\bar{q}_1g_s\sigma\cdot Gq_1\rangle+\frac{1}{6}\left(2m_2\pm m_1\right)\langle\bar{q}_2g_s\sigma\cdot Gq_2\rangle+\right. \\ & +\frac{2}{9}\pi\alpha_s\Big(\langle\bar{q}_1\gamma_\mu\lambda^aq_1\,\bar{q}_1\gamma^\mu\lambda^aq_1\rangle+\langle\bar{q}_2\gamma_\mu\lambda^aq_2\,\bar{q}_2\gamma^\mu\lambda^aq_2\rangle\Big)\pm \\ & \left.\pm 4\pi\alpha_s\langle\bar{q}_1\left\{\begin{array}{c}\gamma_5\\1\end{array}\right\}\lambda^aq_2\,\bar{q}_2\left\{\begin{array}{c}\gamma_5\\1\end{array}\right\}\lambda^aq_1\rangle\right]\,.\end{aligned} \tag{3.8}$$

While $\Pi^-(q^2)$ is relevant for extracting the value of $f_V^\perp$ [4, 12], one gets from $\Pi^+(q^2)$ the $f_A^\perp$ RG-improved sum rules:

$$\begin{aligned}& m_A^2e^{-m_A^2/M^2}L^{-8/(3b)}(f_A^\perp)^2 = \\ & = \frac{1}{8\pi^2}\int_0^{s_0^A}sds\,e^{-s/M^2}\left(1-6\frac{m_1m_2}{s}L^{-8/b}+\frac{\alpha_s}{\pi}\left[\frac{7}{9}+\frac{2}{3}\ln\frac{s}{\mu^2}\right]\right)+\frac{1}{24}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle-\end{aligned}$$

$$
-\left(\frac{1}{2}m_1 - m_2\right)\langle\bar{q}_1 q_1\rangle L^{4/b} - \left(\frac{1}{2}m_2 - m_1\right)\langle\bar{q}_2 q_2\rangle L^{4/b} -
$$

$$
-\frac{1}{M^2}\left[\frac{1}{6}(2m_1 - m_2)\langle\bar{q}_1 g_s\sigma\cdot G q_1\rangle L^{-14/(3b)} + \frac{1}{6}(2m_2 - m_1)\langle\bar{q}_2 g_s\sigma\cdot G q_2\rangle L^{-14/(3b)} - \right.
$$

$$
\left. -\frac{32\pi\alpha_s}{81}\left(\langle\bar{q}_1 q_1\rangle^2 + \langle\bar{q}_2 q_2\rangle^2\right) + \frac{16\pi\alpha_s}{9}\langle\bar{q}_1 q_1\rangle\langle\bar{q}_2 q_2\rangle\right], \tag{3.9}
$$

where $L \equiv \alpha_s(\mu)/\alpha_s(M)$, M is the Borel mass, s_0^A is the threshold for higher resonances, and $f_A^\perp$ depends on the renormalization scale as

$$
f_A^\perp(\mu) = f_A^\perp(\mu_0)\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\frac{4}{3b}}. \tag{3.10}
$$

We then start with the analysis of the $f_A^\perp$ sum rules. We take the experimental mass results for b_1 and h_1 but with larger uncertainties: $m_{b_1} = (1300 \pm 70)\,\text{MeV}$, $m_{h_1} = (1386 \pm 70)\,\text{MeV}$ as inputs to be consistent with QCD sum rule calculations [4]. We also consider the isodoublet strange meson K_{1B} of quantum number 1^1P_1. It should be noted that the real physical states $K_1(1270)$ and $K_1(1400)$ are the mixture of 1^3P_1 and 1^1P_1 states. Following the notations in ref. [13], the relations can be written as

$$
\begin{aligned}
K_1(1270) &= K_{1A}\sin\theta_K + K_{1B}\cos\theta_K\,, \\
K_1(1400) &= K_{1A}\cos\theta_K - K_{1B}\sin\theta_K\,,
\end{aligned} \tag{3.11}
$$

where K_{1A} is the strange mesons of quantum numbers 1^3P_1. The mixing angle θ_K may be close to 45° [3]. In the following study, we take $m_{K_{1B}} = 1370 \pm 70\,\text{MeV}$ [4], where the uncertainty is also enlarged. We obtain the tensor couplings (at scale 1 GeV),

$$
\begin{aligned}
f_{b_1}^\perp &= (180 \pm 10)\,\text{MeV} \qquad \text{for } s_0^{b_1} = (2.6 \pm 0.3)\,\text{GeV}^2\,, \\
f_{h_1}^\perp &= (200 \pm 20)\,\text{MeV} \qquad \text{for } s_0^{h_1} = (3.5 \pm 0.3)\,\text{GeV}^2\,, \\
f_{K_{1B}}^\perp &= (195 \pm 10)\,\text{MeV} \qquad \text{for } s_0^{K_{1B}} = (3.1 \pm 0.3)\,\text{GeV}^2\,,
\end{aligned} \tag{3.12}
$$

where the values of s_0 are determined when the stability of the sum rules is reached within the Borel window $1\,\text{GeV}^2 < M^2 < 1.5\,\text{GeV}^2$. Note that K_{1A} couples to the local axial vector current, instead of the local tensor current; in other words, one has $\langle 0|\bar{q}\gamma_\mu\gamma_5 s|K_{1A}(P,\lambda)\rangle = -if_{K_{1A}}m_{K_{1A}}\epsilon_\mu^{(\lambda)}$. Therefore, according to eq. (3.11), we have $\langle 0|\bar{q}\sigma_{\mu\nu}s|K_1(1270)(P,\lambda)\rangle = if_{K_{1B}}^\perp\cos\theta_K\,\epsilon_{\mu\nu\alpha\beta}\epsilon_{(\lambda)}^\alpha P^\beta$ and $\langle 0|\bar{q}\sigma_{\mu\nu}s|K_1(1400)(P,\lambda)\rangle = -if_{K_{1B}}^\perp\sin\theta_K\,\epsilon_{\mu\nu\alpha\beta}\epsilon_{(\lambda)}^\alpha P^\beta$ with $\bar{q} \equiv \bar{u}, \bar{d}$.

3.3 The Gegenbauer moments for LCDAs $\Phi_\parallel^A$

The LCDAs $\Phi_\parallel^A(u,\mu)$ corresponding to the 1^1P_1 states are defined as[3]

$$
\langle A(P,\lambda)|\bar{q}_1(y)\,\not{z}\gamma_5 q_2(x)|0\rangle = if_A m_A(\epsilon_\lambda^*\cdot z)\int_0^1 dx e^{i(up\cdot y + \bar{u}p\cdot x)}\Phi_\parallel^A(u,\mu)\,, \tag{3.18}
$$

[3] It is known that the coupling of the K_{1B} to the local axial-vector current does not vanish in the isospin

where $z^2 = (y-x)^2 = 0$. $\Phi_\parallel^A(u,\mu)$ can be expanded in a series of Gegenbauer polynomials as given in eq. (2.10). To calculate the Gegenbauer moments of $\Phi_\parallel^A$, we consider the following two-point correlation functions

$$\Pi_{l,\mu\nu} = i\int d^4x e^{iqx}\langle 0|T(\Omega_l^A(x)\, O_{\mu\nu}^\dagger(0)|0\rangle = (zq)^l I_l(q^2)(z_\mu q_\nu - z_\nu q_\mu)\,, \tag{3.19}$$

where, to leading logarithmic accuracy, the relevant multiplicatively renormalizable operator is

$$\Omega_l^A(x) = \sum_{j=0}^{l} c_{l,j}(iz\partial)^{l-j}\bar{q}_2(x)\not{z}\gamma_5\,(iz\overleftrightarrow{D})^j q_1(x)\,, \tag{3.20}$$

with $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu = (\overrightarrow{\partial} + ig_s A^a(x)\lambda^a/2)_\mu - (\overleftarrow{\partial} - ig_s A^a(x)\lambda^a/2)_\mu$, $c_{l,k}$ being the coefficients of the Gegenbauer polynomials such that $C_l^{3/2}(x) = \sum c_{l,k}x^k$. Ω_l^A and $O_{\mu\nu}$ satisfy the following relations:

$$\langle 0|\Omega_l^A(0)|A(P,\lambda)\rangle = -if_A m_A(\epsilon_\lambda\cdot z)(z\cdot P)^l\frac{3(l+1)(l+2)}{2(2l+3)}\,a_l^{\parallel,A}(\mu)\,, \tag{3.21}$$

$$\langle 0|O_{\mu\nu}(0)|A(P,\lambda)\rangle \equiv \langle 0|\bar{q}_2(0)i\sigma_{\mu\nu}\gamma_5 q_1(0)|A(P,\lambda)\rangle = -if_A^\perp(\epsilon_\mu^{(\lambda)}P_\nu - \epsilon_\nu^{(\lambda)}P_\mu)\,. \tag{3.22}$$

The RG-improved sum rules for Gegenbauer moments $a_l^{\parallel,A}$ read

$$\begin{aligned} a_l^{\parallel,A} = &-\frac{1}{m_A f_A f_A^\perp}e^{m_A^2/M^2}\frac{2(2l+3)}{3(l+1)(l+2)}L^{[(4/3)+\gamma_{(l)}^\parallel]/b}\times\\ &\times\Bigg\{\frac{3}{4\pi^2}M^2(1-e^{-s_0^{\parallel,A}/M^2})\left(\int_0^1 d\alpha C_l^{3/2}(2\alpha-1)[m_{q_2}\alpha + m_{q_1}(\alpha-1)]\right)L^{-4/b} -\\ &- C_l^{3/2}(1)(\langle\bar{q}_2q_2\rangle + \langle\bar{q}_1q_1\rangle(-1)^{l+1})L^{4/b} - \end{aligned}$$

---

limit:

$$\langle K_{1B}(P,\lambda)|\bar{s}(0)\gamma_\mu\gamma_5 q(0)|0\rangle = i\bar{f}_{K_{1B}}m_{K_{1B}}\epsilon_\mu^{*(\lambda)}\,, \tag{3.13}$$

i.e., $\bar{f}_{K_{1B}} \neq 0$. Thus one can define the chiral-even leading-twist LCDA as

$$\langle K_{1B}(P,\lambda)|\bar{s}(y)\not{z}\gamma_5 q(x)|0\rangle = i\bar{f}_{K_{1B}}m_{K_{1B}}(\epsilon_\lambda^*\cdot z)\int_0^1 dx e^{i(up\cdot y+\bar{u}p\cdot x)}\bar{\Phi}_\parallel^{K_{1B}}(u,\mu)\,, \tag{3.14}$$

where

$$\bar{\Phi}_\parallel^{K_{1B}}(u,\mu) = 6u(1-u)\left(1+\sum_{l=1}^{\infty}\bar{a}_l^\parallel(\mu)C_l^{3/2}(2u-1)\right). \tag{3.15}$$

Essentially, the above definition is consistent with what we adopted in the present paper, satisfying the following relations

$$a_0^\parallel f_{K_{1B}} = \bar{f}_{K_{1B}}\,,\qquad a_l^\parallel f_{K_{1B}} = \bar{a}_l^\parallel\bar{f}_{K_{1B}}\qquad \text{for } l\geq 1\,, \tag{3.16}$$

and

$$\int_0^1 \Phi_\parallel^{K_{1B}}(u,\mu)du = a_0^\parallel\,,\qquad \int_0^1\bar{\Phi}_\parallel^{K_{1B}}(u,\mu)du = 1\,. \tag{3.17}$$

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$$
\begin{aligned}
&- \left(\frac{1}{3} C_l^{3/2}(1) + 2 C_{l-1}^{5/2}(1)\theta(l-1)\right) \times \\
&\times \frac{\langle \bar{q}_2 g_s \sigma \cdot G q_2\rangle + \langle \bar{q}_1 g_s \sigma \cdot G q_1\rangle (-1)^{l+1}}{M^2} L^{-2/(3b)} - \\
&- \frac{\pi^2}{M^4}\left[\frac{20}{3} C_{l-2}^{7/2}(1)\theta(l-2) + \frac{1}{3} C_{l-1}^{5/2}(1)\theta(l-1))\right] \times \\
&\times \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle [\langle \bar{q}_2 q_2\rangle + \langle \bar{q}_1 q_1\rangle (-1)^{l+1}] L^{4/b} \Bigg\} . \qquad (3.23)
\end{aligned}
$$

The above sum rules for Gegenbauer moments $a_l^{\parallel,A}$ amount to the results in terms of moments $\langle \xi^l_{\parallel,A}\rangle$:

$$
\begin{aligned}
\langle \xi^l_{\parallel,A}\rangle = &- \frac{1}{m_A f_A f_A^\perp} e^{m_A^2/M^2} \times \\
&\times \Bigg\{ \left[\frac{3}{16\pi^2} M^2 \left(\frac{m_{q_2}+m_{q_1}}{l+2} + \frac{m_{q_2}-m_{q_1}}{l+1}\right) - \langle \bar{q}_2 q_2\rangle - \right. \\
&\left. - \frac{2l+1}{3}\frac{\langle \bar{q}_2 g_s \sigma \cdot G q_2\rangle}{M^2} - \frac{\pi^2 l}{9M^4}(4l-3)\left\langle \frac{\alpha_s}{\pi} G^2\right\rangle \langle \bar{q}_2 q_2\rangle \right] + \\
&+ (-1)^{l+1} \left[\frac{3}{16\pi^2} M^2 \left(\frac{m_{q_2}+m_{q_1}}{l+2} - \frac{m_{q_2}-m_{q_1}}{l+1}\right) - \langle \bar{q}_1 q_1\rangle - \right. \\
&\left. - \frac{2l+1}{3}\frac{\langle \bar{q}_1 g_s \sigma \cdot G q_1\rangle}{M^2} - \frac{\pi^2 l}{9M^4}(4l-3)\left\langle \frac{\alpha_s}{\pi} G^2\right\rangle \langle \bar{q}_1 q_1\rangle \right]\Bigg\}, \qquad (3.24)
\end{aligned}
$$

where

$$
\langle \xi^l_{\parallel,A}\rangle_\mu = \int_0^1 dx\,(2x-1)\Phi_\parallel^A(x,\mu)\,. \qquad (3.25)
$$

Five remarks are in order. First, we will simply take $f_A = f_A^\perp(\mu = 1\,\mathrm{GeV})$ in the study since only the products of $f_A a_l^{\parallel,A}$ are relevant. Second, the sum rules obtained from the nondiagonal correlation functions in eq. (3.19) can also determine the sign of $f_A a_l^{\parallel,A}$ relative to $f_A^\perp$. Third, the sum rules for $\langle \xi^l_{\parallel,A}\rangle$ cannot be improved by RG equation since $\langle \xi^l_{\parallel,A}\rangle$ mix with each other even in the one-loop level. For the present case, the RG effects are relatively small compared with the uncertainties of input parameters. Fourth, neglecting the small isospin violation, $a_0^\parallel$ and $a_2^\parallel$ are nonzero only for K_{1B}. Fifth, in the large l limit, the actual expansion parameter is M^2/l for moment sum rules. One can find that, for $l \geq 3$ and fixed M^2, the operator-product-expansion (OPE) series may become divergent. In other words, it is impossible to obtain reliable $a_l^{\parallel,A}$ with $l \geq 3$. In the numerical analysis, we therefore choose the Borel window $(1.0 + l)\,\mathrm{GeV}^2 < M^2 < 2.0\,\mathrm{GeV}^2$ for $a_l^{\parallel,}$ with $l \leq 2$, where the contributions originating from higher resonances and the highest OPE terms are well under control. Using the input parameters given in section 3.1, we obtain the

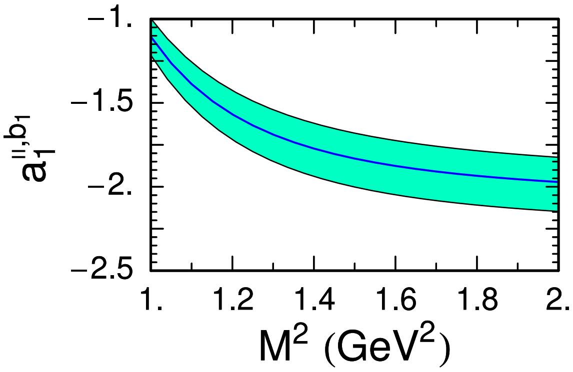

Figure 1: $a_1^{\parallel,b_1}(1\,\mathrm{GeV})$ as a function of the Borel mass squared for $f_{b_1} = f_{b_1}^{\perp}(1\,\mathrm{GeV}) = 180\,\mathrm{MeV}$. The band corresponds to the uncertainties of the input parameters.

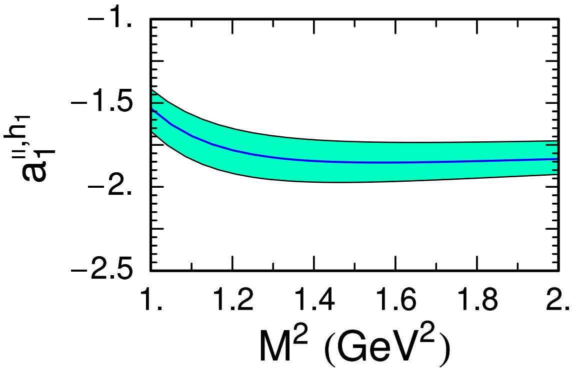

Figure 2: $a_1^{\parallel,h_1}(1\,\mathrm{GeV})$ with $f_{h_1} = f_{h_1}^{\perp}(1\,\mathrm{GeV}) = 200\,\mathrm{MeV}$. Others are the same as figure 1.

Gegenbauer moments, corresponding to $\mu_1 \equiv 1\,\mathrm{GeV}$ and $\mu_2 \equiv 2.2\,\mathrm{GeV}$,

$$\begin{aligned}
a_1^{\parallel,b_1}(\mu_{1[2]}) &= (-1.70 \pm 0.45)\frac{(180\,\mathrm{MeV})^2}{f_{b_1}\, f_{b_1}^{\perp}(\mu_1)}\,, \quad \text{or} \quad (-1.41 \pm 0.37)\frac{(180)(165)\,\mathrm{MeV}^2}{f_{b_1}\, f_{b_1}^{\perp}(\mu_2)}\,, \\
a_1^{\parallel,h_1}(\mu_{1[2]}) &= (-1.75 \pm 0.20)\frac{(200\,\mathrm{MeV})^2}{f_{h_1}\, f_{h_1}^{\perp}(\mu_1)}\,, \quad \text{or} \quad (-1.45 \pm 0.17)\frac{(200)(183)\,\mathrm{MeV}^2}{f_{h_1}\, f_{h_1}^{\perp}(\mu_2)}\,, \\
a_1^{\parallel,K_{1B}}(\mu_{1[2]}) &= (-1.75 \pm 0.25)\frac{(195\,\mathrm{MeV})^2}{f_{K_{1B}}\, f_{K_{1B}}^{\perp}(\mu_1)}\,, \quad \text{or} \quad (-1.45 \pm 0.21)\frac{(195)(179)\,\mathrm{MeV}^2}{f_{K_{1B}}\, f_{K_{1B}}^{\perp}(\mu_2)}\,, \\
a_0^{\parallel,K_{1B}}(\mu_{1[2]}) &= (0.26 \pm 0.06)\frac{(195\,\mathrm{MeV})^2}{f_{K_{1B}}\, f_{K_{1B}}^{\perp}(\mu_1)}\,, \quad \text{or} \quad (0.26 \pm 0.06)\frac{(195)(179)\,\mathrm{MeV}^2}{f_{K_{1B}}\, f_{K_{1B}}^{\perp}(\mu_2)}\,, \\
a_2^{\parallel,K_{1B}}(\mu_{1[2]}) &= (0.13 \pm 0.13)\frac{(195\,\mathrm{MeV})^2}{f_{K_{1B}}\, f_{K_{1B}}^{\perp}(\mu_1)}\,, \quad \text{or} \quad (0.10 \pm 0.10)\frac{(195)(179)\,\mathrm{MeV}^2}{f_{K_{1B}}\, f_{K_{1B}}^{\perp}(\mu_2)}\,. \qquad (3.26)
\end{aligned}$$

The Gegenbauer moments versus the Borel Mass squared are plotted in figures 1–4.

3.4 The Gegenbauer moments for LCDAs $\Phi_\perp^A$

To disentangle the contribution of $\Phi_\perp^A$ from higher twist DAs, it is unavoidable to have an admixture of negative parity vector states in the QCD sum rule study. We consider the

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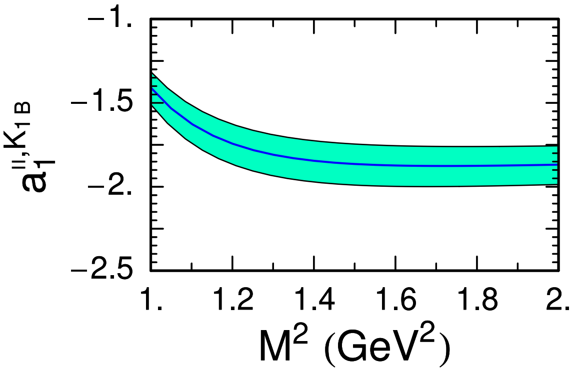

Figure 3: $a_1^{\parallel,K_{1B}}(1\,\mathrm{GeV})$ with $f_{K_{1B}} = f_{K_{1B}}^{\perp}(1\,\mathrm{GeV}) = 195\,\mathrm{MeV}$. Others are the same as figure 1.

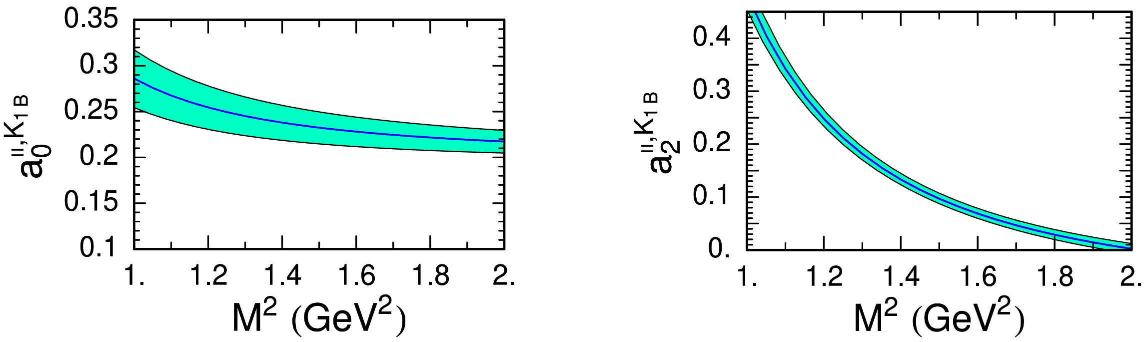

Figure 4: $a_{0,2}^{\parallel,K_{1B}}(1\,\mathrm{GeV})$ with $f_{K_{1B}} = f_{K_{1B}}^{\perp}(1\,\mathrm{GeV}) = 195\,\mathrm{MeV}$. Others are the same as figure 1.

following correlation functions

$$\Pi_l(q^2, qz) = i\int d^4x e^{iqx}\langle 0|T(\Omega_l^{A,T\,\mu}(x)\; O_\mu^\dagger(0)|0\rangle = 2(qz)^{l+2} I_l^\perp(q^2)\,, \tag{3.27}$$

where, to leading logarithmic accuracy, the relevant multiplicatively renormalizable operator is

$$\Omega_l^{A,T\,\mu}(x) = \sum_{j=0}^{l} c_{l,j}(iz\partial)^{l-j}\bar{q}_2(x)\sigma^{\mu\alpha}\gamma_5 z_\alpha\,(iz\overset{\leftrightarrow}{D})^j q_1(x)\,, \tag{3.28}$$

and

$$O_\mu(0) = \bar{q}_1(0)\sigma_{\mu\beta}\gamma_5 z^\beta q_2(0)\,, \tag{3.29}$$

which satisfy the following relations:

$$\begin{aligned}\sum_\lambda \langle 0|\Omega_l^{A,T\,\mu}(0)|A(P,\lambda)\rangle\langle A(P,\lambda)|O_\mu(0)|0\rangle &= 2(f_A^\perp)^2(zP)^{l+2}\frac{3(l+1)(l+2)}{2(2l+3)}\,a_l^{\perp,A}\,,\\ \sum_\lambda \langle 0|\Omega_l^{A,T\,\mu}(0)|V(P,\lambda)\rangle\langle V(P,\lambda)|O_\mu(0)|0\rangle &= 2(f_V^\perp)^2(zP)^{l+2}\frac{3(l+1)(l+2)}{2(2l+3)}\,a_l^{\perp,V}\,.\end{aligned} \tag{3.30}$$

Here V refers to the vector mesons. The Gegenbauer moment sum rules read (c.f. ref. [14])

$$\begin{aligned}
a_l^{\perp,A} = & \frac{1}{(f_A^\perp)^2} e^{m_A^2/M^2} \frac{2(2l+3)}{3(l+1)(l+2)} \times \\
& \times \Bigg\{ \frac{1}{2\pi^2} \frac{\alpha_s}{\pi} M^2 \left(1 - e^{-s_0^{\perp,A}/M^2}\right) \int_0^1 du\, u\bar{u}\, C_l^{3/2}(2u-1) \left(\ln u + \ln \bar{u} + \ln^2 \frac{u}{\bar{u}}\right) + \\
& \quad + \frac{1}{12M^2} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \left(C_l^{3/2}(1) - 2 \right) + \\
& \quad + \frac{1}{M^2} (m_{q_1} \langle \bar{q}_1 q_1 \rangle + m_{q_2} \langle \bar{q}_2 q_2 \rangle) C_l^{3/2}(1) + \\
& \quad + \frac{1}{3M^4} (m_{q_1} \langle \bar{q}_1 g_s \sigma G q_1 \rangle + m_{q_2} \langle \bar{q}_2 g_s \sigma G q_2 \rangle) \left(3C_{l-1}^{5/2}(1) \theta(l-1) + C_l^{3/2}(1) \right) + \\
& \quad + \frac{32\pi\alpha_s}{81M^4} (\langle \bar{q}_1 q_1 \rangle^2 + \langle \bar{q}_2 q_2 \rangle^2) \left(3C_{l-1}^{5/2}(1) \theta(l-1) - C_l^{3/2}(1) \right) - \\
& \quad - \frac{3(l+1)(l+2)}{2(2l+3)} (f_V^\perp)^2 a_l^{\perp,V} e^{-m_V^2/M^2} \Bigg\} \qquad (3.31)
\end{aligned}$$

for even $l \geq 2$, and

$$\begin{aligned}
a_l^{\perp,A} = & \frac{1}{(f_A^\perp)^2} e^{m_A^2/M^2} \frac{2(2l+3)}{3(l+1)(l+2)} \times \\
& \times \Bigg\{ - \frac{1}{M^2} (m_{q_1} \langle \bar{q}_1 q_1 \rangle - m_{q_2} \langle \bar{q}_2 q_2 \rangle) C_l^{3/2}(1) - \\
& \quad - \frac{1}{3M^4} (m_{q_1} \langle \bar{q}_1 g_s \sigma G q_1 \rangle - m_{q_2} \langle \bar{q}_2 g_s \sigma G q_2 \rangle) \left(3C_{l-1}^{5/2}(1) \theta(l-1) + C_l^{3/2}(1) \right) - \\
& \quad - \frac{32\pi\alpha_s}{81M^4} (\langle \bar{q}_1 q_1 \rangle^2 - \langle \bar{q}_2 q_2 \rangle^2) \left(3C_{l-1}^{5/2}(1) \theta(l-1) - C_l^{3/2}(1) \right) - \\
& \quad - \frac{3(l+1)(l+2)}{2(2l+3)} (f_V^\perp)^2 a_l^{\perp,V} e^{-m_V^2/M^2} \Bigg\} \qquad (3.32)
\end{aligned}$$

for odd l.

It should be noted again that for the present Gegenbauer moment sum rules the actual expansion parameter is M^2/l in the large l limit. As a result, for $l \geq 4$ and fixed M^2, the OPE series are convergent slowly or even divergent. In the numerical analysis, we choose the Borel windows (i) $1.1\,\mathrm{GeV}^2 < M^2 < 2.0\,\mathrm{GeV}^2$ for $a_1^{\perp,K_{1B}}$, (ii) $1.2\,\mathrm{GeV}^2 < M^2 < 2.0\,\mathrm{GeV}^2$ for $a_2^{\perp,A}$, and (iii) $1.3\,\mathrm{GeV}^2 < M^2 < 2.0\,\mathrm{GeV}^2$ for $a_3^{\perp,K_{1B}}$ where the contributions originating from higher resonances and the highest OPE terms are under control. We use the parameters given in section 3.1 and the relevant Gegenbauer moments of the LCDAs of vector mesons as inputs. $a_2^{\perp,V}$ at scale 1 GeV are summarized as below [15]:

$$\begin{aligned}
a_2^{\perp,\rho} &= 0.2 \pm 0.1\,, \\
a_2^{\perp,\phi} &= 0.0 \pm 0.1\,, \\
a_2^{\perp,K^*} &= 0.13 \pm 0.08\,. \qquad (3.33)
\end{aligned}$$

$a_1^{\perp,V}$, which is obvious nonzero only for K^*, was studied in ref. [14], where a sign error was found as compared to the original work of Chernyak and Zhitnitsky [16]. The authors in ref. [14] thus concluded that a_1, which refers to a $K^{(*)}$ containing an s quark,[4] should be negative. Nevertheless, Braun and Lenz [17] have analyzed $a_1^{\parallel,K^*}$ and found the result to be 0.10 ± 0.07. They also argued that the sum rule results in ref. [14] for $a_1^{\parallel,K^*}$, $a_1^{\perp,K^*}$ and a_1^K may be unstable owing to many cancellations among OPE terms. Here, if neglecting the K_{1B} effect, we obtain $a_1^{\perp,K^*} = 0.05 \pm 0.02$ and $a_3^{\perp,K^*} = 0.02 \pm 0.02$, where the value of $a_1^{\perp,K^*}$ is consistent with that in ref. [17]. In the present study, we will use $a_1^{\perp,K^*} \simeq a_1^{\parallel,K^*} = 0.10 \pm 0.07$ [17, 15] and $a_3^{\perp,K^*} = 0.02 \pm 0.02$ at the scale 1 GeV. The numerical analysis yields

$$\begin{aligned}
a_2^{\perp,b_1}(\mu_{1[2]}) &= (0.03 \pm 0.19)\left(\frac{180\,\mathrm{MeV}}{f_{b_1}^{\perp}(\mu_1)}\right)^2, \quad \text{or} \quad (0.02 \pm 0.15)\left(\frac{165\,\mathrm{MeV}}{f_{b_1}^{\perp}(\mu_2)}\right)^2, \\
a_2^{\perp,h_1}(\mu_{1[2]}) &= (0.17 \pm 0.29)\left(\frac{200\,\mathrm{MeV}}{f_{h_1}^{\perp}(\mu_1)}\right)^2, \quad \text{or} \quad (0.13 \pm 0.23)\left(\frac{183\,\mathrm{MeV}}{f_{h_1}^{\perp}(\mu_2)}\right)^2, \\
a_2^{\perp,K_{1B}}(\mu_{1[2]}) &= (-0.02 \pm 0.22)\left(\frac{195\,\mathrm{MeV}}{f_{K_{1B}}^{\perp}(\mu_1)}\right)^2, \quad \text{or} \quad = (-0.02 \pm 0.17)\left(\frac{179\,\mathrm{MeV}}{f_{K_{1B}}^{\perp}(\mu_2)}\right)^2, \\
a_1^{\perp,K_{1B}}(\mu_{1[2]}) &= (-0.13 \pm 0.19)\left(\frac{195\,\mathrm{MeV}}{f_{K_{1B}}^{\perp}(\mu_1)}\right)^2, \quad \text{or} \quad (-0.11 \pm 0.17)\left(\frac{179\,\mathrm{MeV}}{f_{K_{1B}}^{\perp}(\mu_2)}\right)^2, \\
a_3^{\perp,K_{1B}}(\mu_{1[2]}) &= (-0.02 \pm 0.08)\left(\frac{195\,\mathrm{MeV}}{f_{K_{1B}}^{\perp}(\mu_1)}\right)^2, \quad \text{or} \quad (-0.01 \pm 0.06)\left(\frac{179\,\mathrm{MeV}}{f_{K_{1B}}^{\perp}(\mu_2)}\right)^2, \qquad (3.34)
\end{aligned}$$

corresponding to the excited state thresholds $s_0^{\perp,b_1} = 2.3 \pm 0.3\,\mathrm{GeV}^2$, $s_0^{\perp,h_1} = 2.7 \pm 0.3\,\mathrm{GeV}^2$ and $s_0^{\perp,K_{1B}} = 2.5 \pm 0.3\,\mathrm{GeV}^2$, respectively, where $\mu_1 \equiv 1\,\mathrm{GeV}$ and $\mu_2 \equiv 2.2\,\mathrm{GeV}$. To consider the vector modes in the excited states, lower magnitudes of thresholds are taken here. The results for Gegenbauer moments are insensitive to the thresholds. In figures 5–8 we plot the Gegenbauer moments as functions of the Borel mass squared, where main uncertainties come from the errors of $a_l^{\perp,V}$.

4. Summary

We have calculated the first few Gegenbauer moments of leading twist light-cone distribution amplitudes of 1^1P_1 mesons using the QCD sum rule technique. The models for light-cone distribution amplitudes depend on the Gegenbauer moments of the truncated conformal expansion. Taking into account $a_{0,1,2}^{\parallel,A}$ and $a_{1,2,3}^{\perp,A}$, we show the light-cone distribution amplitudes in figure 9. $\Phi_\parallel^A(u)$ is asymmetric under $u \leftrightarrow 1-u$ if neglecting SU(3) breaking effects. In particular, we obtain sizable magnitudes for the first Gegenbauer moments of $\Phi_\parallel^A(u)$: $f_A a_1^{\parallel,A}(1\,\mathrm{GeV}) \simeq (-0.23 \sim -0.39)\,\mathrm{GeV}$, which could greatly enhance the longitudinal branching ratios of factorization-suppressed $B \to h_1(1380)K^{(*)}, b_1(1235)K^{(*)}$ modes [5]. Unfortunately, it seems to be impossible to obtain reliable estimates for $f_A a_l^{\parallel,A}$ with $l \geq 3$.

[4] a_1 changes sign for a $K^{(*)}$ involving an $\bar{s}$ quark. In the present paper, we adopt the convention for a_1 referring to $K^{(*)}$ and K_{1B} of containing an s quark.

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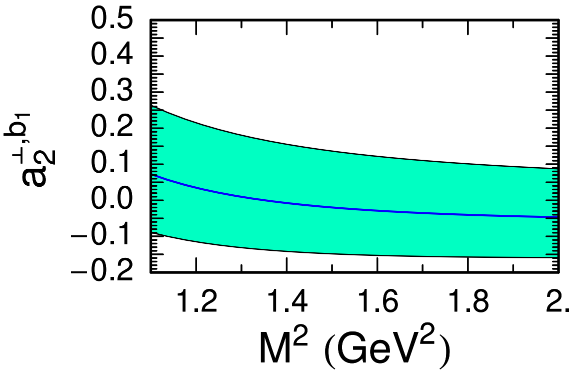

Figure 5: $a_2^{\perp,b_1}(1\,\mathrm{GeV})$ as a function of the Borel mass squared for $f_{b_1}^{\perp}(1\,\mathrm{GeV}) = 180\,\mathrm{MeV}$. The band corresponds to the uncertainties of the input parameters.

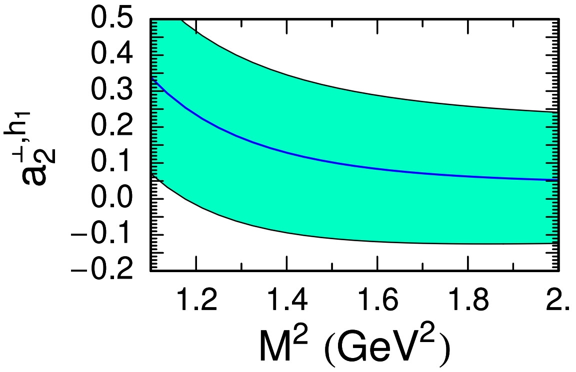

Figure 6: $a_2^{\perp,h_1}(1\,\mathrm{GeV})$ with $f_{h_1}^{\perp}(1\,\mathrm{GeV}) = 200\,\mathrm{MeV}$. Others are the same as figure 5.

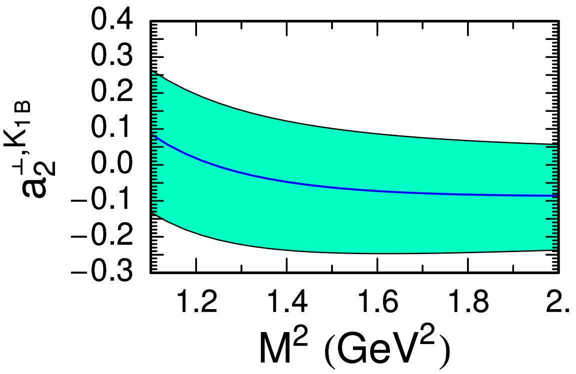

Figure 7: $a_2^{\perp,K_{1B}}(1\,\mathrm{GeV})$ with $f_{K_{1B}}^{\perp}(1\,\mathrm{GeV}) = 195\,\mathrm{MeV}$. Others are the same as figure 5.

Recently, Belle has measured $B^- \to K_1^-(1270)\gamma$ and given an upper bound on $B^- \to K_1^-(1400)\gamma$ [18]. Interestingly, the recent calculations [19, 20] of adopting LCSR (light-cone sum rule) form factors gave too small predictions for $\mathcal{B}(B^- \to K_1^-(1270)\gamma)$ as compared with the data. Since the physical states $K_1(1270)$ and $K_1(1400)$ are the mixture of K_{1A} and K_{1B} which are respectively the pure 1^3P_1 and 1^1P_1 states, the light-cone distribution amplitudes of K_{1A} and K_{1B} are relevant to the results of $B \to K_1(1270)$ and $K_1(1400)$

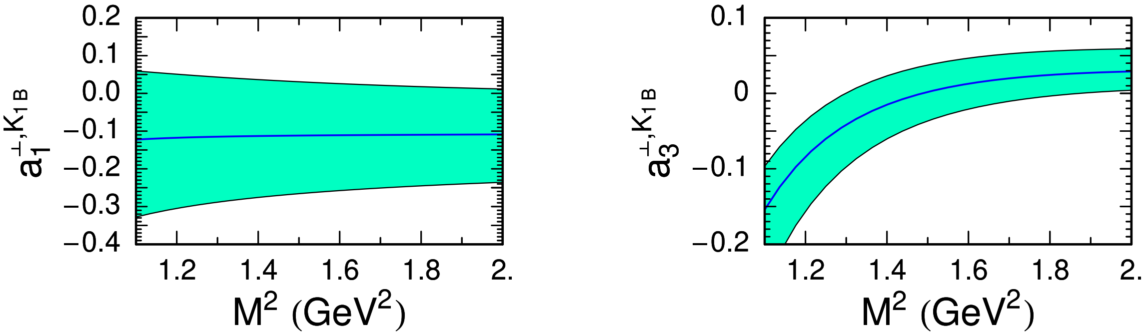

Figure 8: $a_1^{\perp,K_{1B}}(1\,\mathrm{GeV})$ and $a_3^{\perp,K_{1B}}(1\,\mathrm{GeV})$ with $f_{K_{1B}}^{\perp}(1\,\mathrm{GeV}) = 195\,\mathrm{MeV}$. Others are the same as figure 5.

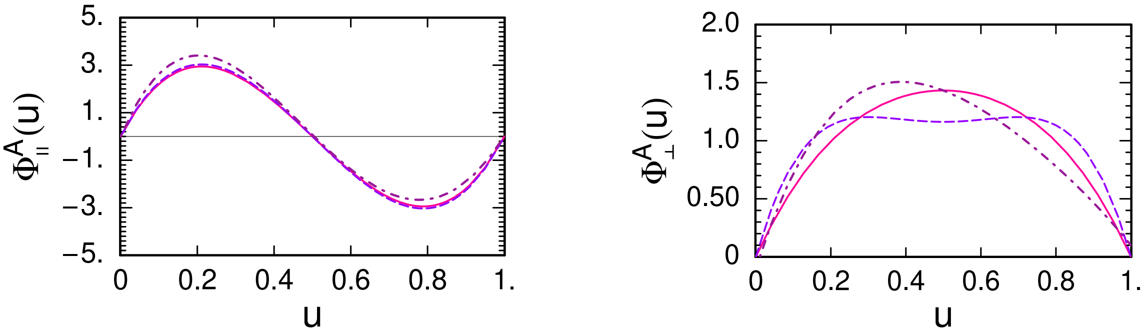

Figure 9: Leading twist light-cone distribution amplitudes at the scale of μ =1 GeV, where the central values of Gegenbauer moments given in eqs. (3.26) and (3.34) are used. u ($\bar{u} \equiv 1 - u$) is the meson momentum fraction carried by the quark (antiquark). The solid, dashed and dot-dashed curves correspond to $b_1(1235), h_1(1380)$ and $\overline{K}_{1B}$, respectively.

transition form factors. It is known that for K_{1B}, $\Phi_{\parallel}$ is antisymmetric, while $\Phi_{\perp}$ is symmetric in the SU(3) limit due to G-parity. Nevertheless, for K_{1A}, $\Phi_{\parallel}$ becomes symmetric, while $\Phi_{\perp}$ is antisymmetric. The above properties were not studied in the literature. These related researches will be published in the near future [21].

Acknowledgments

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